Electromagnetic Surface Modes at Interfaces with Negative Refractive Index make a "Not-Quite-Perfect" Lens.

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Interfaces between media with negative relative refractive index generically support propagating electromagnetic surface polariton modes with large wavenumber. The relation of these modes to a recent prediction by Pendry of "perfect (real) image formation" by a parallel slab of negative-refractive-index material is analyzed. The "perfect image" theory is found to be incomplete without inclusion of a large-wavenumber cutoff that derives from a necessary wavenumber-dependence of the constitutive relations, and which controls the resolution of the image.

Long ago, Veselago [1] noted that if an object could be viewed though a transparent slab of thickness w of a notional material with negative relative refractive index $n = -1$, simple ray optics shows that there is a focused image at a distance $2w$ in front of the object, which is real if the distance between the object and the back surface of the slab is less than w. In general, as seen below, the condition $n = -1$ can be satisfied only for light with a frequency belonging to a discrete set of one or more special frequencies ω^* that characterize an interface that supports "negative refraction".

Ray optics is only valid at lengthscales large compared to the wavelength of light, but recently Pendry [2] reported that if the condition $n = -1$ can be supplemented with the condition of *perfect impedance-matching* between electromagnetic waves in the two media at the special frequency ω^*, the formal solution of Maxwell's equations, with a local effective-medium approximation for the constitutive relations, predicts that a real image formed by light at that frequency is "perfect", in that it reproduces the features of the object at all lengthscales, however small. Pendry describes this counterintuitive prediction from Maxwell's equation as "superlensing"; superficially, his solution appears mathematically correct, but it has been controversial, and various commentators [3, 4, 5, 6] have looked for flaws in his reasoning.

Ruppin [7] has found that, unlike conventional refracting interfaces, interfaces with negative refractive index support surface electromagnetic modes ("surface polaritons"). In this Letter, I point out that Pendry's "perfect image" result, and its limitations, can be understood from a degeneration of these modes in the impedance-matched limit. In the local effective-medium approximation, the surface polaritons become degenerate and dispersionless at the "lensing frequency" ω^*, with no upper limit to their surface wavenumber. It is this unphysical feature that "explains" the "perfect image" prediction, but this does not appear to have been previously explicitly noticed, either by Pendry (though he hints [2] at a connection to "well-defined surface plasmons"), or his

critics, or in Ref. 7. The dispersionless surface modes are clearly a pathology of the approximation which neglects any wavenumber dependence of the constitutive relations. In fact, as with all real matter, the microscopic nature of the lensing medium will provide an "ultraviolet" (large wavenumber) cutoff. This cutoff is relevant for optics *only* in the "superlensing" limit, when it controls the actual resolution of real images.

Since the direction of light rays in the ray-optics limit is the direction of the group velocity of the light, and the component of the wavevector parallel to the surface is conserved during refraction, it is easy to see that a surface with negative relative refractive index in some frequency range is an interface between media which both support propagating long-wavelength electromagnetic waves at those frequencies, but with *group velocities of opposite sign*. The group velocity of electromagnetic waves must be positive in both the low-frequency and high-frequency limits, but can be negative at intermediate frequencies, as shown in Fig.(1); in an isotropic medium, transverse modes with negative group velocity must become degenerate with a finite-frequency longitudinal mode as $k \to 0$.

The Poynting vector $\mathbf{E} \times \mathbf{H}$ of a propagating electromagnetic wave is parallel to the group velocity. Referring to the orthogonal triad of three vectors $(\mathbf{E}, \mathbf{H}, \mathbf{k})$ that characterize propagating electromagnetic waves in an isotropic medium, Veselago [1] introduced the term "*left-handed media*" to describe media with negative group velocity, as opposed to conventional "*right-handed*" media with positive group velocity. (This terminology seems potentially misleading, as the "handedness" it refers to derives from the representation of the magnetic field as an axial vector, not intrinsic chirality of the medium.)

Note that the model spectrum shown in Fig.(1) will only exhibit dissipation at the frequencies ω_{T1} and ω_{T2}. Some of the commentators [4, 6] have suggested that Pendry's calculation must fail in practice because it omits absorption effects, which, because the constitutive relations are frequency-dependent, must be present at some frequencies because of the Kramers-Krönig relation. However, this does *not* require dissipation in the
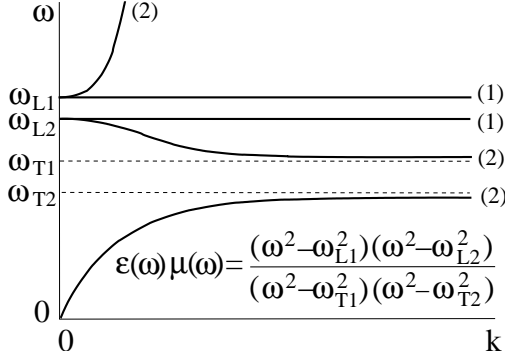
FIG. 1: In this model spectrum, the photon couples to the transverse components of two polarization modes (one electric, one magnetic) in a material. The propagating modes at low and high frequencies are of the usual so-called "right-handed" type, with a positive group velocity, but are "left-handed" in the frequency range $\omega_{T1} < \omega < \omega_{L2}$, where the group velocity is negative. A surface has a negative refraction index in a frequency range where it is an interface between bulk regions with opposite signs of the group velocity.

frequency range of interest. In fact, if the "left-handed" medium can be treated as loss-free, it must be a periodic structure, so a microscopic lengthscale is provided by its lattice spacing. If the material is a perfectly-periodic "photonic crystal", and possesses only one propagating mode (of each polarization) in the frequency range of interest, it can be treated as non-dissipative to the extent that non-linearity is negligible.

In an isotropic medium with frequency-dependent but *local* (wavenumber-independent) constitutive relations, the spectrum of electromagnetic waves predicted by the effective Maxwell equations is given by $c^2k^2 = \omega^2\epsilon(\omega)\mu(\omega)$, where ϵ and μ are the dielectric constant and relative permeability. If waves in the two media have opposite group velocities, there will be a particular frequency ω^* at which the wavelengths of propagating waves in the two media coincide: $\lambda_1(\omega^*) = \lambda_2(\omega^*) = 2\pi/k^*$; the condition $n = -1$ for a flat interface to produce a focused image is only realized at this special frequency. The group velocity v at this frequency is given by the expansion in $\delta\omega = \omega - \omega^*$:

$$\omega^2\epsilon(\omega)\mu(\omega) = c^2k^{*2} + 2c^2k^*v^{-1}\delta\omega + O(\delta\omega)^2. \quad (1)$$

Let the interface be the plane $z = 0$. Following Ruppin [7], I look for an "S-polarized" interface mode

$$B^z(x,y,z) = B_0^z e^{\kappa_1 z} e^{i(k_\parallel x - \omega t)}, \quad z \le 0,$$
$$= B_0^z e^{-\kappa_2 z} e^{i(k_\parallel x - \omega t)}, \quad z \ge 0, \quad (2)$$

where κ_1 and κ_2 are both positive; B^z couples to H^x and E^y, and all are continuous at the interface: $k_\parallel E^y = \omega B^z$, and $k_\parallel \mu_0 H^x = i\alpha B^z$, where [7]

$$\alpha = \frac{\kappa_1}{\mu_1} = -\frac{\kappa_2}{\mu_2}. \quad (3)$$

This only has a solution when μ_1/μ_2 is negative. Since by assumption $\epsilon_1\mu_1$ and $\epsilon_2\mu_2$ are both positive, ϵ_2/ϵ_1 is also negative, and the interface has negative refractive index. The source of these fields is an oscillating transverse surface polarization current

$$J^y(x,y) = J_0^y e^{i(k_\parallel x - \omega t)}, \quad (4)$$
$$k_\parallel \mu_0 J_0^y = i\alpha(\mu_1 - \mu_2)B_0^z. \quad (5)$$

The condition giving the frequency of the mode comes from combining (3) with

$$c^2\kappa_1^2 = c^2k_\parallel^2 - \omega^2\epsilon_1\mu_1,$$
$$c^2\kappa_2^2 = c^2k_\parallel^2 - \omega^2\epsilon_2\mu_2. \quad (6)$$

For frequencies close to ω^*, the expansion (1) can be used: for small positive $\delta k_\parallel = k_\parallel - k^*$, $\delta\omega/\delta k_\parallel \to v$, as $\delta k_\parallel \to 0^+$, where

$$v = \frac{v_1 v_2 (\kappa_1^2 - \kappa_2^2)}{(v_1\kappa_1^2 - v_2\kappa_2^2)}, \quad (7)$$

where v_1 and v_2 are the group velocities in the two media at frequency ω^*, which have opposite signs.

This result is easy to understand: the group velocity of the surface mode is a weighted average of the bulk group velocities of the two media, with a larger weight given to the velocity in the medium with smaller κ, into which the fields penetrate deeper. In the special case $\mu_1 = -\mu_2$, $\kappa_1 = \kappa_2$, the competition between the two media is exactly balanced, and v vanishes; in this limit the predicted frequency of the mode becomes perfectly dispersionless with $\omega(k_\parallel) = \omega^*$ for all $k_\parallel > k^*$. This corresponds to perfect impedance matching at frequency ω^*: $\mu_1(\omega^*)/\epsilon_1(\omega^*) = \mu_2(\omega^*)/\epsilon_2(\omega^*)$, when no reflection of incident propagating waves with $k_\parallel < k^*$ will occur.

There is a second ("P-polarized") mode, deriving from longitudinal surface polarization currents:

$$J^x(x,y) = J_0^x e^{i(k_\parallel x - \omega t)}, \quad (8)$$
$$D^z(x,y,z) = D_0^z e^{\kappa_1' z} e^{i(k_\parallel x - \omega t)}, \quad z \le 0,$$
$$= D_0^z e^{-\kappa_2' z} e^{i(k_\parallel x - \omega t)}, \quad z \ge 0, \quad (9)$$

where $\omega(\mu_2 - \mu_1)D_0^z = k_\parallel J_0^x$, $k_\parallel H^y = -\omega D^z$, and $k_\parallel \epsilon_0 E^x = i\alpha' D^z$, with [7]

$$\alpha' = \frac{\kappa_1'}{\epsilon_1} = -\frac{\kappa_2'}{\epsilon_2}. \quad (10)$$

Because $\epsilon_1(\omega^*)/\epsilon_2(\omega^*) = \mu_2(\omega^*)/\mu_1(\omega^*)$, when one of these interface modes has positive group velocity along the interface, the other group velocity is negative. The spectrum is schematically depicted in Fig.(2).

It is now instructive to examine the coupling between the two sets of modes on the opposite faces of a slab of width d of a medium 2 embedded in medium 1. The consistency condition (3) for the (B^z, H^x, E^y) slab polariton
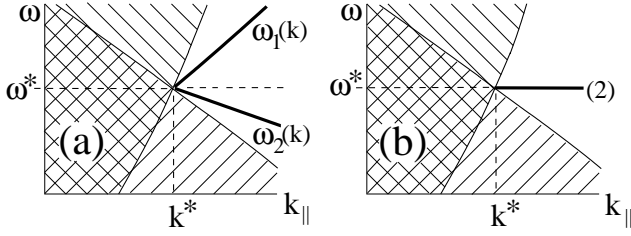
FIG. 2: (a): Generic spectrum of electromagnetic modes that propagate along a negative-refractive-index interface with surface wavenumber $k_\parallel > k^*$: there are two surface modes, respectively with positive and negative group velocity; the shaded regions of the $(\omega, k_\parallel)$ plane indicate where either one or both of the media supports propagating bulk modes, and ω^* is the special frequency at which waves have the same wavelength $\lambda^* = 2\pi/k^*$ in both media. (b): The degenerate spectrum predicted by the local Maxwell equations in the impedance-matched limit where "perfect lens" behavior has been predicted: the two surface modes become degenerate at the frequency ω^* for all $k_\parallel > k^*$. Such exactly-dispersionless degenerate surface modes are an artifact of the approximation which neglects wavenumber-dependence of the constitutive relations of the "left-handed" medium.

modes becomes [8]

$$\frac{\kappa_1}{\mu_1} = -\frac{\kappa_2}{\mu_2}\left(\tanh(\kappa_2 d/2)\right)^{\pm 1}, \qquad (11)$$

where the $\pm$ distinguishes the modes which have symmetric $(+)$ and antisymmetric $(-)$ flux $B^z(z)$. When $(k_\parallel - k^*)d \gg 1$, $\kappa_2 d$ is very large, and the splitting between the symmetric and antisymmetric combinations of the modes on the two faces is very small, proportional to $\exp(-k_\parallel d)$. In the other limit, they become the $n = 0$ and $n = 1$ bands of modes confined to the slab, that emerge from the edge of the continuum of exterior propagating modes with initial group velocity v_1 at $k_\parallel \simeq k^* - n^2\pi^2/2k^*d^2$, $n \geq 0$ (the $n \geq 2$ bands remain within the region of the $(\omega, k_\parallel)$-plane where the slab medium has propagating modes). Similar considerations apply to the (D^z, E^x, H^y) modes, but with μ replaced by ϵ. The predicted spectrum is shown schematically in Fig.(3). There is a single band-crossing of the surface modes (allowed because "S" and "P" polarizations do not mix) exactly at the frequency ω^*, when $\kappa_1 = \kappa_2 = \kappa_0$, at a surface wavenumber $k_0 > k^*$ given by:

$$k_0^2 = k^{*2} + \kappa_0^2, \quad e^{-\kappa_0 d} = \gamma^2 < 1, \qquad (12)$$

where γ is the ratio

$$\gamma = \left(\frac{\mu_2(\omega^*) + \mu_1(\omega^*)}{\mu_2(\omega^*) - \mu_1(\omega^*)}\right) = \left(\frac{\epsilon_1(\omega^*) + \epsilon_2(\omega^*)}{\epsilon_1(\omega^*) - \epsilon_2(\omega^*)}\right) \qquad (13)$$

(the two expressions for γ are equivalent because $\epsilon_1\mu_1 = \epsilon_2\mu_2$ at the frequency ω^*). Note that $k_0 \to \infty$ in the impedance-matched "perfect lens" limit where $\gamma = 0$.
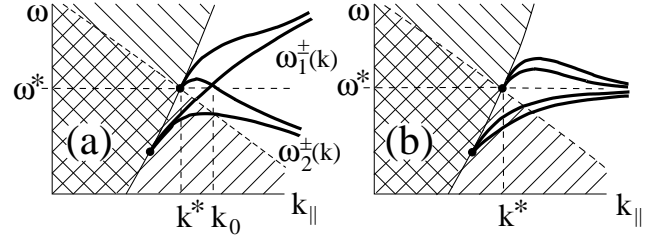

FIG. 3: (a): Generic spectrum of the coupled electromagnetic surface modes of a flat slab of "left-handed" medium, with finite thickness d, embedded in a standard "right-handed" medium, calculated assuming local (wavenumber-independent) constitutive relations. The splitting between the even and odd combinations of the surface modes becomes exponentially small for $k_\parallel$ large. An allowed band crossing occurs exactly at the frequency ω^*, at $k_\parallel = k_0$ (see text). (b): Predicted spectrum in the "perfect lens" limit (perfect impedance matching). The band-crossing point k_0 recedes to $k_\parallel = \infty$, and for large $k_\parallel$ the surface mode frequencies differ from the "perfect lens" frequency ω^* by exponentially small splittings proportional to $\exp(-k_\parallel d)$.

I now examine the solution of Maxwell's equations for the steady-state radiation field of an object illuminated with radiation at the special frequency ω^*. The sources of the radiation field are the oscillating currents in the object that are excited by the illuminating field. I will assume the source current distribution is restricted to the region $z \leq 0$, and that the object is viewed though a slab of "left-handed" medium that is present in the region $0 < z_1 < z < z_2$, where $z_2 - z_1 = d > 0$. The source can be resolved into transverse Fourier components $\mathbf{k}_\parallel$ in the x and y coordinates, and the radiation field of each Fourier component computed separately. Consider a Fourier component with $k_\parallel^2 - k^{*2} = \kappa^2 > 0$, which produces an evanescent field in the region $z > 0$:

$$B^z = B_0 e^{i(k_\parallel x - \omega^* t)} F(z, \gamma),$$

$$k_\parallel E^y = \omega^* B^z, \quad -ik_\parallel \mu_0 H^x = \mu^{-1}\frac{\partial B^z}{\partial z}, \qquad (14)$$

$$D^z = D_0 e^{i(k_\parallel x - \omega^* t)} F(z, -\gamma),$$

$$k_\parallel H^y = -\omega^* D^z, \quad -ik_\parallel \epsilon_0 E^x = \epsilon^{-1}\frac{\partial D^z}{\partial z}, \qquad (15)$$

where (using continuity of H^x and E^x at the interfaces)

$$F(z, \gamma) = e^{-\kappa z} + \alpha e^{-\kappa|z-z_1|} + \beta e^{-\kappa|z-z_2|}, \qquad (16)$$

$$\alpha = \left(\frac{\gamma + e^{-2\kappa d}}{\gamma^2 - e^{-2\kappa d}}\right) e^{-\kappa z_1}, \qquad (17)$$

$$\beta = -\left(\frac{\gamma + 1}{\gamma^2 - e^{-2\kappa d}}\right) e^{-\kappa z_2}. \qquad (18)$$

When presented in this form, the solution of Maxwell's equations has a simple physical interpretation: $F(z, \gamma)$ is the sum of the evanescent radiation field of the object, driven directly by the illumination, plus the radiation

fields of the induced surface polarization currents of the slab, driven by the radiation field of the object. The amplitudes of the two surface currents diverge when $k_\parallel = k_0$ $(\exp -2\kappa d = \gamma^2)$, when the driving frequency ω^* is exactly resonant with the coupled surface modes. This resonance recedes to $k_\parallel = \infty$ as the impedances are tuned to the "perfect lens" limit $\gamma = 0$.

When $\gamma = 0$, the field of the small surface polarization current on the left interface at z_1 exactly cancels the field of the object for $z > z_1$, while for $z < z_1$, it cancels the field of the second polarization current on the right interface at z_2. The field observed for $z > z_1$ is then just the radiation field of the surface polarization current at z_2. If $z_1 < d$, this current has an amplitude that, as $k_\parallel \to \infty$, *grows exponentially* $\propto \exp k_\parallel (d - z_1)$ relative to the strength of the source radiation field. Since the object was taken to be to the left of the plane $z = 0$, the condition $z_1 < d$ is precisely the condition that a real image of some part of the object can be formed.

The predicted exponentially-large amplification in the large-wavenumber limit limit occurs because *the difference between the coupled surface mode frequencies and the driving frequency ω^* becomes exponentially small* as $k_\parallel \to \infty$ (see Fig.(3b). It is a pathology of Pendry's solution that was noted in Ref. 6; however, while those authors recognized that, for $|z - z_2| < d - z_1$, this exponential amplification leads to an "ultra-violet" (large wavenumber) divergence of the expression for the predicted radiation field of a point object at $z = 0$, they drew the incorrect conclusion that this divergence implied that the evanescent radiation field of such a point object cannot penetrate the slab. In fact, the divergence will be always controlled by the short-distance cutoff provided by the physical nature of the interface; this may either be the maximum wavenumber of a surface polarization current (*e.g.*, a surface Brillouin zone boundary), or a wavenumber at which k-dependent constitutive relations move the surface mode frequencies away from near-resonance with the driving frequency ω^*.

Pendry's solution is only strictly valid without a cutoff in the case $z_1 > d$, but remains non-singular in the marginal case $z_1 = d$, where the image of a source at $z = 0$ is neither real nor virtual, but is exactly *on* the right surface $z = z_2$ of the slab. In this limit, there is an interesting interpretation of Pendry's formal result: the replacement $\epsilon \to -\epsilon$, $\mu \to -\mu$ is equivalent to the transformation $(\mathbf{B}, \mathbf{D}, \mathbf{E}, \mathbf{H}) \to (-\mathbf{B}, -\mathbf{D}, \mathbf{E}, \mathbf{H})$, which in turn is equivalent to time-reversal of the source-free Maxwell equations. The predicted "perfect image" formed by monochromatic radiation with frequency ω^* is then analogous to the "spin-echo" after a π-pulse in magnetic resonance: the radiation field from a source at $z = 0$ first propagates a distance d along the z-axis in the normal medium, with dispersion of its $\mathbf{k}_\parallel$-Fourier-components, then this dispersion is exactly reversed by subsequent propagation through an equal thickness d of the "time-reversing" medium to refocus the field at the "perfect image" point. However, this intriguing interpretation is spoiled by the inequivalence between a physical "left-handed" medium and an idealized "time-reversed vacuum": the equivalence is only valid at a single frequency ω^*, and as $k \to 0$ the frequency of "left-handed light" approaches a finite value ω_L (see Fig.(1)); also, the medium will provide a large-wavenumber cutoff.

In summary, I have shown Pendry's [2] controversial "superlens" theory is incomplete, in that it fails to explicitly include a large-momentum cutoff, which is needed to regularize the theory when it describes a real image. The high Fourier components of the image are produced by the near-fields of surface polariton modes [7, 8] which in the "superlens" limit are exponentially close to resonance at the special lensing frequency ω^*. The microscopic structure of the surface of the lensing medium will determine this cutoff, which is unspecified in a local-effective medium approximation, and will limit the resolution of the image. While "not-quite-perfect", it seems that this resolution can in principle be engineered to be significantly smaller than the wavelength of the illuminating radiation, without violating any fundamental physical principles.

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